Probability logic as a rationality framework

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Workshop on non-classical logic and probabilistic reasoning
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- Computational level problem description (in the sense of Marr, 1982): reason to an interpretation of the premises and to draw a rational conclusion. This requires to
  1. make any implicit assumptions and logical relations explicit,
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- Long term goal: development of a unified, normative and descriptively adequate theory of human reasoning under uncertainty

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  - probability logic: mental probability logic, etc.

Rationality Framework: Coherence-based probability logic

- Coherence (subjective probability)
  - de Finetti, and {Capotorti, Coletti, Gilio, Holzer, Lad, Regazzini, Rigo, Sanfilippo, Scozzafava, Vantaggi, Walley, . . . }
  - probability as a degree of belief
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  - logical operations on conditional events (without triviality)
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- Probability logic
  - uncertain argument forms
  - deductive transmission of the uncertainties from the premises to the conclusion

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Mental probability logic: Experimental evidence

- Uncertain argument forms
  - monotonic versus nonmonotonic arguments: most people draw coherent inferences and understand that monotonic arguments are probabilistically non-informative (Pfeifer & Kleiter, 2005, 2010; Pfeifer & Tulkki, 2017b)
  - conditional syllogisms: e.g., probabilistic modus ponens (*from* $p(C|A)$ *and* $p(A)$ *infer* $p(C)$) is easier for people compared to the probabilistic modus tollens (*from* $p(C|A)$ *and* $p(\neg C)$ *infer* $p(\neg A)$) (Pfeifer & Kleiter, 2007, 2009)

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- Argumentation
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- Conditionals
  - Paradoxes of the material conditional (e.g., *from* C *infer* $A \to C$), are probabilistically non-informative and hence blocked in mental probability logic. This matches the data **(Pfeifer & Kleiter, 2011; Pfeifer, 2014)**.
  - Observed response tendencies in tasks involving negations in samples of Westerners did not differ from a Japanese sample: a first step towards cross-cultural comparisons **(Pfeifer & Yama, 2017)**

Probabilistic truth table task (Evans, Handley, & Over, 2003; Oberauer & Wilhelm, 2003)

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\begin{array}{rcl}
p(A \wedge C) & = & x_1 \\
p(A \wedge \neg C) & = & x_2 \\
p(\neg A \wedge C) & = & x_3 \\
p(\neg A \wedge \neg C) & = & x_4 \\
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p(\text{If } A, \text{ then } C) & = & ?
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Sample conclusion candidates:

- $p(A \wedge C) = x_1$
- $p(C|A) = x_1/(x_1 + x_2)$
- $p(A \supset C) = x_1 + x_3 + x_4$

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- more than half of the responses are consistent with $p(C|A)$
- many responses are consistent with $p(A \wedge C)$

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- sample task material (sides of a die): ○ □ ■ ○ ○ ○

  Iterated version: interpretation shifts to $p(C|A)$ (Fugard, Pfeifer, Mayerhofer, & Kleiter, 2011; Kleiter, Fugard, & Pfeifer, 2018)

Generalised probabilistic truth table tasks: results

Conditional probability $p(C|A)$ is the best predictor for beliefs in conditionals, even if

- the premises probabilities are imprecise **(Pfeifer, 2013a; Pfeifer & Tulkki, 2017a)**, like:

○	□	■	○	○	?

$$p(\text{circle} \wedge \text{white}) \in [\tfrac{3}{6}, \tfrac{4}{6}]$$
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- . . . a counterfactual, i.e., fact (not A) plus *If A were the case, C would be the case* **(see, e.g., Pfeifer & Stöckle-Schobel, 2015; Pfeifer & Tulkki, 2017a)**

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- data are also robust for conditionals which violate Δp:

$$\Delta p = p(C|A) - p(C|\neg A) > 0$$

Comments on inferentialist accounts of conditionals

- Inferentialist accounts claim that some inferential connection between the antecedent A and the consequent C is needed in order to build a belief in the conditional $A \to C$, which could, for example be
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  This is inconsistent with inferentialist accounts. Participants' judgments are also independent of Δp.
- Centering (*from* A *and* C *infer* $A \to C$) holds in probability logic (Sanfilippo et al., 2018), but not in inferentialist accounts (e.g. Douven, 2016, p. 40). Most people endorse centering (e.g., Cruz et al., 2016; Pfeifer & Tulkki, 2017b), which is not consistent with inferentialism.

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The basic connexive intuition is covered by the observation that for any event A, with $\neg A \neq \perp$, event $A|\neg A$ is

$$P(A|\neg A) = 0,$$

where 0 is the only coherent value (Pfeifer & Sanfilippo, 2021).

Example 1: Connexive principles (Wansing, 2020)

- Aristotle's Thesis (AT): $\neg(\neg A \to A)$
- Aristotle's Thesis' (AT)$'$: $\neg(A \to \neg A)$
- Abelard's First Principle (AB): $\neg((A \to B) \wedge (A \to \neg B))$
- Aristotle's Second Thesis (AS): $\neg((A \to B) \wedge (\neg A \to B))$
- Boethius' Thesis (BT): $(A \to B) \to \neg(A \to \neg B)$
- Boethius' Thesis' (BT)$'$: $(A \to \neg B) \to \neg(A \to B)$
- Reversed Boethius' Thesis (RBT): $\neg(A \to \neg B) \to (A \to B)$
- Reversed Boethius' Thesis' (RBT)$'$: $\neg(A \to B) \to (A \to \neg B)$
- Boethius Variation 3 (B3): $(A \to B) \to \neg(\neg A \to B)$
- Boethius Variation 4 (B4): $(\neg A \to B) \to \neg(A \to B)$

Example 1: Valid in probability logic? (Pfeifer & Sanfilippo, 2021)

- Aristotle's Thesis (AT): $\neg(\neg A \to A)$ ✓
- Aristotle's Thesis' (AT)′: $\neg(A \to \neg A)$ ✓
- Abelard's First Principle (AB): $\neg((A \to B) \wedge (A \to \neg B))$ ✓
- Aristotle's Second Thesis (AS): $\neg((A \to B) \wedge (\neg A \to B))$ ×
- Boethius' Thesis (BT): $(A \to B) \to \neg(A \to \neg B)$ ✓
- Boethius' Thesis' (BT)′: $(A \to \neg B) \to \neg(A \to B)$ ✓
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Further theoretical developments

Example 2: We developed a coherence-based probability semantics for Aristotelian syllogisms (Gilio, Pfeifer, & Sanfilippo, 2016; Pfeifer & Sanfilippo, 2018, 2019, submitted), which

- respects the logical structure of the argument forms,
- uses very weak existential import assumptions (weaker than assuming $p(S) > 0$),
- allows for dealing with generalised quantifiers,
- constitutes a bridge to nonmonotonic reasoning, and
- explains the logical relations among the basic syllogistic sentence types in terms of the probabilistic square and hexagon of opposition (Pfeifer & Sanfilippo, 2017a, 2017b).

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Example 3: Generalisation of Modus Ponens and explanation why participants' responses in the indicative/counterfactual conditions in the probabilistic truth table task do not differ.

From modus ponens to generalised modus ponens

	Modus ponens	Generalised modus ponens
(Categorical premise)	A	$A\|H$
(Conditional premise)	If A, then C	If $A\|H$, then C
(Conclusion)	C	C

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Sample instantiation (Gibbard, 1981, p. 237):

$\overbrace{\textit{The cup breaks if dropped}}^{A|H}$.

If $\overbrace{\textit{the cup breaks if dropped}}^{A|H}$, then $\overbrace{\textit{the cup is fragile}}^{C}$.

Therefore, $\overbrace{\textit{the cup is fragile}}^{C}$.

Generalised Probabilistic MP (Sanfilippo, Pfeifer, & Gilio, 2017)

Generalised modus ponens	Generalised probabilistic modus ponens
$A\|H$	$p(A\|H) = x$
If $A\|H$, then C	$\mathbb{P}(C\|(A\|H)) = y$
C	$? \leq p(C) \leq ?$

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If $A \mid H$, then C	$\mathbb{P}(C \mid (A \mid H)) = y$
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What does the conditional premise mean? It is a conditional random quantity.

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In betting terms, $\mu = \mathbb{P}[C|(A|H)]$ represents the amount you agree to pay, with the proviso that you will receive the quantity (Gilio & Sanfilippo, 2013b):

$$C|(A|H) = \begin{cases} 1, & \text{if } A \wedge H \wedge C \text{ true,} \\ 0, & \text{if } A \wedge H \wedge \neg C \text{ true,} \\ \mu, & \text{if } \neg A \wedge H \text{ true,} \\ x + \mu(1-x), & \text{if } \neg H \wedge C \text{ true,} \\ \mu(1-x), & \text{if } \neg H \wedge \neg C \text{ true.} \end{cases}$$

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Since $(C|A)|H \neq C|(A \wedge H)$, the Import-Export Principle does not hold. Thus, Lewis' first triviality result (1976) is avoided (Gilio & Sanfilippo, 2014).

Generalised modus ponens (Sanfilippo, Pfeifer, & Gilio, 2017, Theorem 5, p. 487)

Generalised modus ponens	Generalised probabilistic modus ponens
$A\|H$	$p(A\|H) = x$
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How do we propagate the uncertainty from the premises to the conclusion?

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How do we propagate the uncertainty from the premises to the conclusion?

Theorem

Given any coherent assessment (x, y) on $\{A|H, C|(A|H)\}$, with A, C, H logically independent, but $A \neq \perp$ and $H \neq \perp$. The conclusion $p(C)$ is coherent iff

$$xy \leq p(C) \leq xy + 1 - x$$

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Theorem

Given any coherent assessment (x, y) on $\{A|H, C|(A|H)\}$, with A, C, H logically independent, but $A \neq \bot$ and $H \neq \bot$. The conclusion $p(C)$ is coherent iff

$$xy \leq p(C) \leq xy + 1 - x\,,$$

which are just the same probability propagation rules as in the non-nested probabilistic modus ponens, i.e., *from $p(A) = x$ and $p(C|A) = y$ infer* $xy \leq P(C) \leq xy + 1 - x$.

Explanation of why responses in the indicative/counterfactual conditions *should* not differ

Theorem **(see, e.g. Gilio & Sanfilippo, 2013a)**:

$$\overbrace{\textit{Prevision}\ \underbrace{[(C|A)|\overbrace{\neg A}^{\text{fact}}]}_{\text{cond. random quantity}}}^{\text{belief in counterfactual}} = \overbrace{\textit{Probability}\ (\underbrace{C|\overbrace{A}^{\text{assumed}}}_{cond.event})}^{\text{belief in indicative conditional}}.$$

Table of contents

Concluding remarks

- Coherence-based probability logic provides a rich and unified rationality framework, including
  - meanings of conditionals (embracing indicative conditionals, simple and nested, and counterfactuals) and their behaviour in uncertain argument forms
  - nonmonotonic reasoning
  - conditional syllogisms
  - measuring argument strength
  - paradoxes of the material conditional
  - connexive principles
  - Aristotelian syllogisms
  - Probabilistic square and hexagon of opposition
- Choice of the interpretation of probability is crucial (e.g., managing zero-probability antecedents, etc.)
- Focus should be on uncertainty propagation

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