

Probability logic as a rationality framework

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What is *Mental Probability Logic*?

What is a rationality framework?

Brief overview on experimental evidence

Further theoretical developments (3 examples)

Concluding remarks

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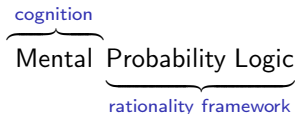
cognition

Mental Probability Logic

rationality framework

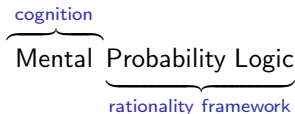
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 - ▶ reasoning competence
 - ▶ epistemic reasoning and argumentation problems

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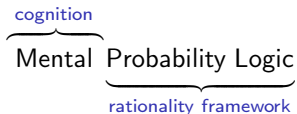
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- ▶ Assumptions:
 - ▶ Everyday life reasoning is based on incomplete information and uncertain premises, conclusions are defeasible.
 - ▶ People interpret the uncertainty of *If A, then C* by $p(C|A)$.
 - ▶ **Rationality framework**: coherence-based probability logic

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 - ▶ **Rationality framework**: coherence-based probability logic
- ▶ Computational level **problem description** (in the sense of Marr, 1982): reason to an interpretation of the premises and to draw a rational conclusion. This requires to
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 2. assign uncertainty to the premises, and
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 3. transmit the uncertainty from the premises to the conclusion.
- ▶ Long term **goal**: development of a unified, normative and descriptively adequate theory of human reasoning under uncertainty

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- ▶ allows psychologists to evaluate the data

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Rationality Framework: Coherence-based probability logic

- ▶ Coherence (subjective probability)
 - ▶ de Finetti, and {Capotorti, Coletti, Gilio, Holzer, Lad, Regazzini, Rigo, Sanfilippo, Scozzafava, Vantaggi, Walley, ... }
 - ▶ probability as a **degree of belief**
 - ▶ in betting terms, a probability assessment is **coherent** if and only if in any finite combination of n bets, it cannot happen that the values of the random gain are all positive or all negative (**no Dutch Book**)
 - ▶ complete algebra is **not required**

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$$\frac{p(A \wedge C)}{p(A)}, \quad \text{provided that } p(A) > 0$$

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In the coherence approach, conditional probability, $p(C|A)$, is **primitive** and properly managed even if $p(A) = 0$

- ▶ **zero probabilities** are exploited to reduce the complexity
- ▶ **imprecision**
- ▶ **logical operations** on conditional events (without triviality)
- ▶ bridges to **possibility**, **fuzzy sets**, **nonmonotonic reasoning**, ...

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- ▶ uncertain argument forms
 - ▶ **deductive** transmission of the uncertainties from the premises to the conclusion

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- ▶ Uncertain argument forms

- ▶ **monotonic versus nonmonotonic** arguments: most people draw coherent inferences and understand that monotonic arguments are probabilistically non-informative (Pfeifer & Kleiter, 2005, 2010; Pfeifer & Tulkki, 2017b)
- ▶ **conditional syllogisms**: e.g., probabilistic modus ponens (*from $p(C|A)$ and $p(A)$ infer $p(C)$*) is easier for people compared to the probabilistic modus tollens (*from $p(C|A)$ and $p(\neg C)$ infer $p(\neg A)$*) (Pfeifer & Kleiter, 2007, 2009)

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- ▶ Argumentation

- ▶ Formal **measure of argument strength** (Pfeifer, 2007, 2013b), which
- ▶ allows for rationally reconstructing data observed in the **Ellsberg Paradox** (Pfeifer & Pankka, 2017)

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▶ Conditionals

- ▶ **Paradoxes of the material conditional** (e.g., *from C infer $A \rightarrow C$*), are probabilistically non-informative and hence blocked in mental probability logic. This matches the data (Pfeifer & Kleiter, 2011; Pfeifer, 2014).
- ▶ Observed response tendencies in tasks involving negations in samples of Westerners did not differ from a Japanese sample: a first step towards **cross-cultural comparisons** (Pfeifer & Yama, 2017)

Probabilistic truth table task (Evans, Handley, & Over, 2003; Oberauer & Wilhelm, 2003)

$$\begin{array}{rcl} p(A \wedge C) & = & x_1 \\ p(A \wedge \neg C) & = & x_2 \\ p(\neg A \wedge C) & = & x_3 \\ p(\neg A \wedge \neg C) & = & x_4 \\ \hline p(\text{If } A, \text{ then } C) & = & ? \end{array}$$

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Sample conclusion candidates:

- ▶ $p(A \wedge C) = x_1$
- ▶ $p(C|A) = x_1 / (x_1 + x_2)$
- ▶ $p(A \supset C) = x_1 + x_3 + x_4$

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- ▶ $p(C|A) = x_1 / (x_1 + x_2) = .50$
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Main results:


- ▶ more than half of the responses are consistent with $p(C|A)$
- ▶ many responses are consistent with $p(A \wedge C)$

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- ▶ sample task material (sides of a die): 

Iterated version: interpretation shifts to $p(C|A)$ (Fugard, Pfeifer, Mayerhofer, & Kleiter, 2011; Kleiter, Fugard, & Pfeifer, 2018)

Generalised probabilistic truth table tasks: results

Conditional probability $p(C|A)$ is the **best predictor** for beliefs in conditionals, even if

- ▶ the premises probabilities are **imprecise** (Pfeifer, 2013a; Pfeifer & Tulkki, 2017a), like:



$$p(\text{circle} \wedge \text{white}) \in \left[\frac{3}{6}, \frac{4}{6}\right]$$

$$p(\text{circle} \wedge \neg \text{white}) \in \left[0, \frac{1}{6}\right]$$

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- ▶ \dots a **counterfactual**, i.e., **fact (not A)** plus *If A were the case, C would be the case* (see, e.g., Pfeifer & Stöckle-Schobel, 2015; Pfeifer & Tulkki, 2017a)

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- ▶ data are also **robust** for conditionals which violate Δp :

$$\Delta p = p(C|A) - p(C|\neg A) > 0$$

Comments on inferentialist accounts of conditionals

- ▶ **Inferentialist accounts** claim that some inferential connection between the antecedent A and the consequent C is needed in order to build a belief in the conditional $A \rightarrow C$, which could, for example be
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- ▶ **Centering** (*from A and C infer $A \rightarrow C$*) holds in probability logic (Sanfilippo et al., 2018), but not in inferentialist accounts (e.g. Douven, 2016, p. 40). Most people endorse centering (e.g., Cruz et al., 2016; Pfeifer & Tulkki, 2017b), which is not consistent with inferentialism.

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Conditionals of the form

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Under the material conditional interpretation of conditionals, however, it holds that:

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The basic connexive intuition is covered by the observation that for any event A , with $\neg A \neq \perp$, event $A|\neg A$ is

$$P(A|\neg A) = 0,$$

where 0 is the only coherent value (Pfeifer & Sanfilippo, 2021).

Example 1: Connexive principles (Wansing, 2020)

- Aristotle's Thesis (AT): $\neg(\neg A \rightarrow A)$
- Aristotle's Thesis' (AT)': $\neg(A \rightarrow \neg A)$
- Abelard's First Principle (AB): $\neg((A \rightarrow B) \wedge (A \rightarrow \neg B))$
- Aristotle's Second Thesis (AS): $\neg((A \rightarrow B) \wedge (\neg A \rightarrow B))$
- Boethius' Thesis (BT): $(A \rightarrow B) \rightarrow \neg(A \rightarrow \neg B)$
- Boethius' Thesis' (BT)': $(A \rightarrow \neg B) \rightarrow \neg(A \rightarrow B)$
- Reversed Boethius' Thesis (RBT): $\neg(A \rightarrow \neg B) \rightarrow (A \rightarrow B)$
- Reversed Boethius' Thesis' (RBT)': $\neg(A \rightarrow B) \rightarrow (A \rightarrow \neg B)$
- Boethius Variation 3 (B3): $(A \rightarrow B) \rightarrow \neg(\neg A \rightarrow B)$
- Boethius Variation 4 (B4): $(\neg A \rightarrow B) \rightarrow \neg(A \rightarrow B)$

Example 1: Valid in probability logic? (Pfeifer & Sanfilippo, 2021)

- Aristotle's Thesis (AT): $\neg(\neg A \rightarrow A)$ ✓
- Aristotle's Thesis' (AT)': $\neg(A \rightarrow \neg A)$ ✓
- Abelard's First Principle (AB): $\neg((A \rightarrow B) \wedge (A \rightarrow \neg B))$ ✓
- Aristotle's Second Thesis (AS): $\neg((A \rightarrow B) \wedge (\neg A \rightarrow B))$ ✗
- Boethius' Thesis (BT): $(A \rightarrow B) \rightarrow \neg(A \rightarrow \neg B)$ ✓
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Further theoretical developments

Example 2: We developed a coherence-based probability semantics for [Aristotelian syllogisms](#) (Gilio, Pfeifer, & Sanfilippo, 2016; Pfeifer & Sanfilippo, 2018, 2019, submitted), which

- ▶ respects the logical structure of the argument forms,
- ▶ uses very weak existential import assumptions (weaker than assuming $p(S) > 0$),
- ▶ allows for dealing with generalised quantifiers,
- ▶ constitutes a bridge to nonmonotonic reasoning, and
- ▶ explains the logical relations among the basic syllogistic sentence types in terms of the probabilistic square and hexagon of opposition (Pfeifer & Sanfilippo, 2017a, 2017b).

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- Example 3: [Generalisation of Modus Ponens](#) and [explanation](#) why participants' responses in the indicative/counterfactual conditions in the [probabilistic truth table task](#) do not differ.

From modus ponens to generalised modus ponens

	Modus ponens	Generalised modus ponens
(Categorical premise)	A	$A H$
(Conditional premise)	If A , then C	If $A H$, then C
(Conclusion)	C	C

Generalised Probabilistic MP (Sanfilippo, Pfeifer, & Gilio, 2017)

Generalised modus ponens	Generalised probabilistic modus ponens
$A H$	$p(A H) = x$
If $A H$, then C	$\mathbb{P}(C (A H)) = y$
C	$? \leq p(C) \leq ?$

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In **betting terms**, $\mu = \mathbb{P}[C|(A|H)]$ represents the amount you agree to pay, with the proviso that you will receive the quantity (Gilio & Sanfilippo, 2013b):

$$C|(A|H) = \begin{cases} 1, & \text{if } A \wedge H \wedge C \text{ true,} \\ 0, & \text{if } A \wedge H \wedge \neg C \text{ true,} \\ \mu, & \text{if } \neg A \wedge H \text{ true,} \\ x + \mu(1 - x), & \text{if } \neg H \wedge C \text{ true,} \\ \mu(1 - x), & \text{if } \neg H \wedge \neg C \text{ true.} \end{cases}$$

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Since $(C|A)|H \neq C|(A \wedge H)$, the **Import-Export Principle** does not hold. Thus, Lewis' first triviality result (1976) is avoided (Gilio & Sanfilippo, 2014).

Generalised modus ponens (Sanfilippo, Pfeifer, & Gilio, 2017, Theorem 5, p. 487)

Generalised modus ponens	Generalised probabilistic modus ponens
$A H$	$p(A H) = x$
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How do we propagate the uncertainty from the premises to the conclusion?

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Theorem

Given any coherent assessment (x, y) on $\{A|H, C|(A|H)\}$, with A, C, H logically independent, but $A \neq \perp$ and $H \neq \perp$. The conclusion $p(C)$ is coherent iff

$$xy \leq p(C) \leq xy + 1 - x$$

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which are just the **same probability propagation rules** as in the non-nested probabilistic modus ponens, i.e., from $p(A) = x$ and $p(C|A) = y$ infer $xy \leq P(C) \leq xy + 1 - x$.

Explanation of why responses in the indicative/counterfactual conditions *should* not differ

Theorem (see, e.g. Gilio & Sanfilippo, 2013a):

$$\underbrace{\text{belief in counterfactual}}_{\text{Prevision } [(C|A) | \overbrace{\neg A}^{\text{fact}}]} = \underbrace{\text{belief in indicative conditional}}_{\text{Probability } (C | \overbrace{A}^{\text{assumed}})} .$$

cond. random quantity
cond.event

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Concluding remarks

- ▶ Coherence-based probability logic provides a rich and unified rationality framework, including
 - ▶ meanings of conditionals (embracing indicative conditionals, simple and nested, and counterfactuals) and their behaviour in uncertain argument forms
 - ▶ nonmonotonic reasoning
 - ▶ conditional syllogisms
 - ▶ measuring argument strength
 - ▶ paradoxes of the material conditional
 - ▶ connexive principles
 - ▶ Aristotelian syllogisms
 - ▶ Probabilistic square and hexagon of opposition
- ▶ Choice of the interpretation of probability is crucial (e.g., managing zero-probability antecedents, etc.)
- ▶ Focus should be on uncertainty propagation

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